VARIATION OF WELL PRESSURE AND FLOW RATE IN A POROUS-FRACTURED RESERVOIR

R. I. Medvedskii

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 2, pp. 151-157, 1968

ABSTRACT: Upper and lower bounds are determined for pressure variation with constant flow and for flow variation at constant pressure. A solution of the problem based on the averaging method is also presented. Graphs of the characteristic theoretical pressure variation and experimental curves for the Kyurovdag pool are given.

1. The development of hydrodynamic methods for investigating wells drilled into porous-fractured reservoirs is based on the solution of the following two principal problems: 1) to find the pressure variation $\Delta p(t)$ when the steady state of the bed is disturbed by a sharp but constant change in the well flow rate Δq_0; 2) to find the change in flow rate $\Delta q(t)$ when the steady state is disturbed by a sharp but constant change in pressure Δp_0.

Let the function U(r, t) denote the deviation of the pressure in the joints of the disturbed bed from the steady-state value. In porous-fractured media, with negligibly small joint compressibility, the function U(r, t) satisfies the differential equation

$$\varkappa\nabla^2 U - \varkappa\tau\frac{\partial\nabla^2 U}{\partial t} = \frac{\partial U}{\partial t}, \tag{1.1}$$

derived in [1]. Here, $\nabla^2 U = (1/r)\partial/\partial r(r\,\partial U/\partial r)$, r is the radial distance from the well into the bed, t is the time, $\varkappa$ is the piezoconductivity, τ is a characteristic of the medium with the dimension of time, the so-called lag. The lag is related to the permeability of the joints k, the mass transfer coefficient of the joints, and the blocks α by the expression: $\tau = k/\varkappa\alpha$.

Mathematically, problem (1) can be formulated as follows: to solve Eq. (1.1) with the initial and boundary conditions [2]:

$$U(r, t=0) = 0, \quad -\frac{2\pi kh}{\mu}\left(r\frac{\partial U}{\partial r}\right)_{r=r_1} = \Delta q_0,$$

$$U(r=\infty, t) = 0.$$

Here, kh/μ is the hydroconductivity of the bed and r_1 is the radius of the well. The second boundary condition has been introduced in the given form because the first phase of seepage is being considered.

The Laplace transform of the pressure change

$$\Delta p(s) = \frac{\mu}{2\pi kh}\Delta q_0\left[\frac{K_0[\zeta(\tau)]}{s\zeta(\tau)K_1[\zeta(\tau)]}\right]$$

$$\left(\zeta(\tau) = \left[\frac{1}{m}\frac{s}{1+\tau s}\right]^{1/2}, \quad m = \frac{\varkappa}{r_1^2}\right).$$

Here, s is the Laplace transform parameter, K_0 and K_1 are the standard notations for the Macdonald functions of zeroth and first order, respectively.

Let $F(m\tau, mt)$ denote the inverse transform of the expression in square brackets from (1.2). The presence of the factor m in front of τ and t is dictated by the similarity theorem of Laplace operational calculus [3]. Henceforth, this will be called the pressure function of the bed and will, for simplicity, be denoted by $F_\tau(mt)$. As follows from (1.1) or (1.2), that, at $\tau = 0$, the function $F_0(mt)$ is nothing other than the pressure function of a porous bed [4], the explicit expression being well known (as a function of time).

Problem (2) is formulated mathematically as follows: to solve Eq. (1.1) with the initial and boundary conditions

$$U(r, t=0) = 0, \quad U(r=r_1, t) = \Delta p_0,$$

$$U(r=\infty, t) = 0.$$

Here again, the form of the second boundary condition is determined by the fact that we are concerned with the first phase of seepage.

In Laplace transforms we have

$$\Delta q(s) = \frac{2\pi kh}{\mu}\Delta p_0\left[\frac{\zeta(\tau)K_1[\zeta(\tau)]}{sK_0[\zeta(\tau)]}\right]. \tag{1.3}$$

Let $\Phi(m\tau, mt)$ denote the inverse transform of the expression within the square brackets of (1.3). Here, the presence of the factor m in front of τ and t follows from the same similarity theorem. Henceforth this function will be called the well flow function and will be denoted by $\Phi_\tau(mt)$. Obviously, $\Phi_0(mt)$ is nothing other than the familiar flow function of a porous bed.

In accordance with the definition of $F_\tau(mt)$, when $\tau = 0$ it follows, from (1.2), that

$$\frac{K_0[\zeta(0)]}{s\zeta(0)K_1[\zeta(0)]} = \int_0^\infty e^{-s\sigma}F_0(m\sigma)\,d\sigma =$$

$$= \int_0^\infty e^{-\zeta^2(0)m\sigma}F_0(m\sigma)\,d\sigma. \tag{1.4}$$

Substituting $\zeta(\tau)$ for $\zeta(0)$, after simple transformations we obtain

$$\frac{K_0[\zeta(\tau)]}{s\zeta(\tau)K_1[\zeta(\tau)]} = \int_0^\infty \frac{m\zeta^2(\tau)}{s}e^{-\zeta^2(\tau)m\sigma}F_0(m\sigma)\,d\sigma. \tag{1.5}$$

Let the inverse transform of $ms^{-1}\zeta^2(\tau)\exp[-\zeta^2(\tau)m\sigma$ be $P(t, \sigma)$. Then, in accordance with [3],

$$P(t, \sigma) = \tau^{-1}\exp[-(t+\sigma)\tau^{-1}]\,I_0(2\sqrt{t\sigma/\tau^2}), \tag{1.6}$$

where I_0 is a modified Bessel function of order zero.

Going over to inverse transforms on both sides of (1.5), we obtain the integral representation of the pressure function

$$F_\tau(mt) = \int_0^\infty P(t, \sigma)F_0(m\sigma)\,d\sigma, \tag{1.7}$$

Similarly,

$$\Phi_\tau(mt) = \int_0^\infty P(t, \sigma)\Phi_0(m\sigma)\,d\sigma. \tag{1.8}$$

Since [5] Jensen inequalities can be applied to them, integral representations (1.7) and (1.8) can be used to find the upper and lower bounds of the pressure and flow functions of a porous-fractured bed. As will be shown below, the upper and lower bounds thus obtained differ only slightly, especially at large times. The actual method of obtaining them can be used to solve a fairly wide range of problems in subsurface hydraulics.

2. In this section, we establish certain results required in subsequent constructions.

The function $F_0(mt)$ can be approximately represented as

$$F_0(t_0) \approx \ln(1+\sqrt{4t_0/\pi}) + \frac{1}{2}\ln\frac{\pi}{\gamma}\left(\frac{\sqrt{4t_0/\pi}}{1+\sqrt{4t_0/\pi}}\right)^3; \tag{2.1}$$

$t_0 = mt$; $\gamma = 1.781\ldots$ is Euler's constant.

Since the principal terms of the asymptotic expansion of the function $F_0(t_0)$ and approximation (2.1) coincide both at small and large values of the dimensionless parameter t_0, for these values of t_0, the error of (2.1) is negligible. As may be seen from Table 1, for other values of t_0 the error is also small and does not exceed 0.5%.

In [4], it was shown that, over the entire range of variation of the dimensionless parameter, $\Phi_0(t_0)$ is represented with an error of not more than 1.2% by the formula

$$\Phi_0(t_0) = [\ln(1+\sqrt{\pi t_0})]^{-1}. \tag{2.2}$$

In view of the negligible error of approximations (2.1) and (2.2) in applied problems, it is preferable not to differentiate between them and the exact expressions for the pressure and flow functions of a porous bed, which are inconvenient both for calculation purposes and for investigation.

Table 1

mt	$\sim F_0$	F_0	ε, %
0.3	0.501	0.503	−0.3
0.5	0.615	0.616	−0.3
1.0	0.797	0.802	−0.5
2.5	1.105	1.101	+0.1
5.0	1.365	1.362	+0.1
10.0	1.657	1.651	+0.4
20.0	1.965	1.960	+0.3
25.0	2.066	2.067	−0.1
100.0	2.730	2.723	+0.1
200.0	3.068	3.064	+0.2
400.0	3.420	3.406	+0.5
1000.0	3.864	3.86	+0.1

We consider the functions

$$f(x)=\ln(1+ax)+\frac{1}{2}\ln\frac{\pi}{\gamma}\left(\frac{ax}{1+ax}\right)^3,\quad a=\sqrt{\frac{4m}{\pi}},$$
$$\varphi(x)=\ln^{-1}(1+bx),\quad b=\sqrt{\pi m}. \tag{2.3}$$

The following assertions are directly verifiable: $f(x)$ and $\varphi(x^{-1})$ are convex upwards at all $x \geq 0$ and the functions $f(e^x)$ and $\varphi(e^x)$ are convex downwards for $-\infty < x < \infty$.

Let p_n be a system of positive numbers, where $1 \leq n \leq N$, and let Tx_n be an averaging operator defined as

$$Tx_n=\left(\sum_1^N p_nx_n\right)\left(\sum_1^N p_n\right)^{-1},\quad T^{-1}x_n=\frac{1}{Tx_n}.$$

Then, for any bounded system of positive numbers, we have the Jensen inequalities following from the convexity of the previously introduced [5] functions:

$$Tf(x_n)<f(Tx_n),\quad Tf(\exp y_n)>f(\exp Ty_n),$$
$$T\varphi(1/y_n)<\varphi(T^{-1}y_n),\quad T\varphi(\exp y_n)>\varphi(\exp Ty_n).$$

Setting $y_n = \ln x_n$ in the second and fourth of these inequalities and $y_n = 1/x_n$ in the third, we obtain

$$f(\exp T\ln x_n)<Tf(x_n)<f(Tx_n),$$
$$\varphi(\exp T\ln x_n)<T\varphi(x_n)<\varphi(1/Tx_n).$$

Let $M(t, \sigma)$ be an arbitrary function of the two variables t and σ, increasing monotonically with an increase in σ such that the Stieltjes integrals introduced below exist. Moreover, let the averaging operator $Tf(t)$ be given by

$$Tf(t)=\left(\int_\alpha^\beta f(\sigma)\,dM(t,\sigma)\right)\left(\int_\alpha^\beta dM(t,\sigma)\right)^{-1}\quad(\beta>\alpha\geqslant 0). \tag{2.4}$$

Then, as it is easy to show by passing to the limit in (2.4), we have the following inequalities:

$$f(\exp T\ln\sqrt{t})<Tf(\sqrt{t})<f(T\sqrt{t}),$$
$$\varphi(\exp T\ln\sqrt{t})<T\varphi(\sqrt{t})<\varphi(1/T\sqrt{t}). \tag{2.5}$$

We introduce the notation

$$\sqrt{t_a}=T\sqrt{t},\quad \sqrt{t_g}=\exp T\ln\sqrt{t},\quad \sqrt{t_h}=1/T\sqrt{t^{-1}}. \tag{2.6}$$

Obviously, the functionals $t_a^{1/2} > t_g^{1/2} > t_h^{1/2}$ are the arithmetic, geometric, and harmonic means, respectively, of the function $t^{1/2}$.

Using the fact that $f(t^{1/2}) = F_0(mt)$ and $\varphi(t^{1/2}) = \Phi_0(mt)$, from (2.5) we obtain the following inequalities:

$$F_0(mt_g)<TF_0(mt),<F_0(mt_a),$$
$$\Phi_0(mt_g)<T\Phi_0(mt)<\Phi_0(mt_h), \tag{2.7}$$

which form the basis of our method of determining the upper and lower bounds.

3. In order to determine the upper and lower bounds of the pressure and flow functions, in (2.6) and (2.7) we must set

$$dM(t,\sigma)=P(t,\sigma)\,d\sigma,\ \alpha=0,\ \beta=\infty,$$

where the function $P(t, \sigma) \geq 0$ is found from (1.6).

The integrals in (2.6) are evaluated in accordance with the following model:

$$\int_0^\infty P(t,\sigma)\,d\sigma=\int_0^\infty\frac{m\zeta^2(\tau)}{s}\exp[-m\zeta^2(\tau)\sigma]\,d\sigma=\frac{1}{s},$$
$$\text{i.e., }\int_0^\infty P(t,\sigma)\,d\sigma=1,\text{ etc.} \tag{3.1}$$

After the necessary computations, we obtain

$$\sqrt{t_a}=\frac{1}{2}\sqrt{\pi\tau}\left[2\int_0^x e^{-\sigma}J_0(\sigma)\,\partial\sigma+e^{-x}J_0(x)\right],$$
$$\sqrt{t_h}=\sqrt{\tau/\pi}\,\frac{e^x}{J_0(x)}$$
$$\sqrt{t_g}=\sqrt{2\tau}\exp\{{}^1\!/_2[\ln x-\mathrm{Ei}(-2x)]\}. \tag{3.2}$$

Here,

$$x=\frac{t}{2\tau},\quad -\mathrm{Ei}(-x)=\int_x^\infty\frac{e^{-\sigma}}{\sigma}\,d\sigma.$$

From (1.7), (1.8), (3.1), and the definition of the averaging operator it follows that

$$TF_0(mt)=F_\tau(mt),\quad T\Phi_0(mt)=\Phi_\tau(mt).$$

Hence, from inequalities (2.7) we obtain the following estimates:

$$F_0(mt_g)<F_\tau(mt)<F_0(mt_a),$$
$$\Phi_0(mt_g)<\Phi_\tau(mt)<\Phi_0(mt_h), \tag{3.3}$$

where t_α, t_g, and t_h are found from (3.2).

In Table 2, values of the dimensionless functions $x_a = t_a/2\tau$, $x_g = t_g/2\tau$, and $x_h = t_h/2\tau$ are compared with the upper and lower bounds F_+, Φ_+ and F_-, Φ_- of the pressure and flow functions, respectively, for $\tau_0 = m\tau = 10$ and $\tau_0 = 100$.

As may be seen from Table 2, the maximum difference between the upper and lower bounds for $F_\tau(mt)$ does not exceed 10% at $\tau_0 = m\tau = 10$ and 7% at $\tau_0 = 100$, so that, as τ_0 increases, the discrepancy

Table 2

x	x_a	x_g	x_h	$m\tau = 10$		$m\tau = 100$		$m\tau = 10$		$m\tau = 100$	
				F_+	F_-	F_+	F_-	Φ_+	Φ_-	Φ_+	Φ_-
0	0.39	0.28	0.16	1.55	1.41	2.61	2.44	0.7	0.61	0.416	0.388
1	1.46	1.05	0.73	2.14	1.96	3.26	3.06	0.49	0.455	0.322	0.305
2	2.5	2.01	1.63	2.38	2.28	3.52	3.41	0.476	0.398	0.286	0.28
3	3.5	3.0	2.68	2.55	2.48	3.68	3.61	0.38	0.372	0.268	0.264
4	4.5	4.0	3.7	2.67	2.62	3.81	3.75	0.358	0.355	0.257	0.255
5	5.5	5.0	4.7	2.78	2.73	3.90	3.86	0.345	0.34	0.25	0.249
6	6.5	6.0	5.75	2.84	2.80	4.0	3.94	0.334	0.332	0.245	0.242
7	7.5	7.0	6.7	2.92	2.86	4.07	4.02	0.328	0.324	0.24	0.238
8	8.5	8.0	7.75	2.98	2.95	4.11	4.10	0.318	0.316	0.235	0.234
9	9.5	9.0	8.8	3.03	3.01	4.15	4.14	0.312	0.311	0.232	0.231
10	10.5	10.0	9.75	3.09	3.06	4.21	4.2	0.308	0.306	0.229	0.228

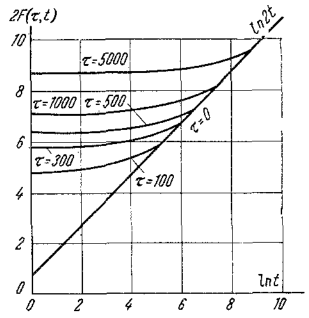

Fig. 1

diminishes. Moreover, the maximum difference between the upper and lower bounds for $\Phi_\tau(mt)$ does not exceed 13% at $\tau_0 = 10$ or 5% at $\tau_0 = 100$, this difference also decreasing as τ_0 increases.

It is clear from (3.2) that, when $\tau_0 = 0$, $t_a = t_g = t_h = t$, so that the difference between the upper and lower bounds (3.3) also diminishes the direction of decreasing lag. Hence the maximum difference corresponds to some small τ_0, above and below which the estimates approach.

Of estimates (3.3), the simplest will be the lower, which is also suitable for use as an approximate evaluation of the pressure and flow functions. Figures 1 and 2 show graphs of the lower bounds as a function of the dimensionless time $t_0 = mt$ for various values of the dimensionless lag.

At $\tau_0 = m\tau \geq 100$, with a small error that decreases with time, the lower bound of the pressure function can be represented in the form

$$F_0(mt_g) \approx {}^1/_2\,[\ln\,({}^4/\gamma)\;mt - \mathrm{Ei}\,(-t/\tau)].$$

This approximation of the function $F_\tau(mt)$ was obtained in [7] by another means.

An investigation of the curves presented in Fig. 1 reveals that, except for the case $\tau = 0$, the pressure function has a discontinuity at $t = 0$, which is greater, the greater the lag. Moreover, at relatively large $m\tau$ the pressure function curve has a clearly expressed horizontal plateau whose length depends directly on the lag, after which it tends fairly rapidly to the common asymptote $\ln(4/\gamma)mt/2$.

Thus, a pressure step following a sharp change in flow rate (up or down) is a characteristic sign ("indicator") of a porous-fractured bed, whose joints have negligibly small compressibility. Curves of this kind are encountered in connection with the investigation of certain injection wells in the Kyurovdag field of the Azerbaidzhanian SSR (Fig. 3).

4. In order to obtain an approximate expression for the pressure function of a porous-fractured bed we will use the averaging method [6], which has proved suitable for solving many problems of nonstationary seepage in a porous bed. Although its formal application to the present problem leads to serious difficulties, these can be avoided as described below.

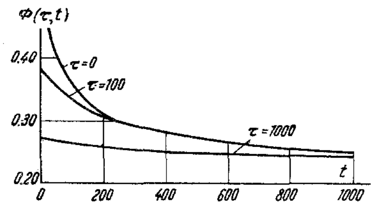

Fig. 2

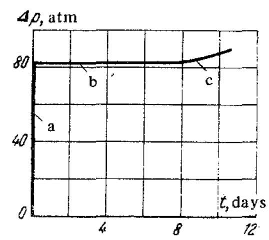

Fig. 3

We introduce the dimensionless variables

$$\lambda = \frac{t}{\tau}, \quad \xi = \frac{r}{\sqrt{\varkappa\tau}}, \quad V(\xi, \lambda) = \frac{2\pi kh}{\mu}\frac{U(r, t)}{\Delta q_0},$$

and for simplicity assume that the radius of the well is negligibly small.

Then, problem (1) reduces to the following: to solve the equation

$$\nabla^2 V = \frac{\partial V}{\partial\lambda} - \frac{\nabla\partial^2 V}{\partial\lambda}, \tag{4.1}$$

with the initial and boundary conditions

$$V(\xi, \lambda = 0) = 0,$$

$$-(\xi\,\partial V/\partial\xi)_{\xi=0} = 1, \; V(\xi = \infty, \lambda) = 0.$$

We introduce the function

$$V(\xi, \lambda) = W(\xi, \lambda) + \partial W(\xi, \lambda)/\partial\lambda, \tag{4.2}$$

which, in accordance with [1], is proportional to the pressure in the porous blocks, whereas the function V is proportional to the pressure in the joints. It is directly verifiable that the function W satisfies the differential equation

$$\nabla^2 W = dW/\partial\lambda - \partial\nabla^2 W/\partial\lambda, \tag{4.3}$$

which must be solved for the initial and boundary conditions

$$W(\xi, \lambda = 0) = 0, \quad (\xi\partial W/\partial\xi)_{\xi=0} + \frac{\partial}{\partial\lambda}(\xi\partial W/\partial\xi)_{\xi=0} = -1,$$

$$W(\xi = \infty, \lambda) = 0. \tag{4.4}$$

If

$$q(\lambda) = -\xi\,\partial W/\partial\xi, \tag{4.5}$$

we can rewrite the first of the boundary conditions of (4.4) in the form

$$\partial q/\partial\lambda + q = 1. \tag{4.6}$$

Solving this differential equation on the assumption that

$$q(0) = 0, \tag{4.7}$$

which is consistent with the physical conditions of the problem, we find that

$$q(\lambda) = 1 - e^{-\lambda}. \tag{4.8}$$

Following the method of averaging, we replace the right-hand side of (4.3) with the mean:

$$F(\lambda) = \left\langle \frac{\partial W}{\partial\lambda} - \frac{\partial\nabla^2 W}{\partial\lambda} \right\rangle. \tag{4.9}$$

In (4.9), the corner brackets denote the averaging operation, applicable to any function $f(\xi)$ defined for $\xi > 0$,

$$\langle f(\xi)\rangle = \frac{2}{R_1}\int_0^R f(\xi)\,\xi d\xi. \tag{4.10}$$

Thus, comparing (4.9) with (4.3), we have $\nabla^2 W = F(\lambda)$ in the region $0 \leq \xi \leq R(\lambda)$ with moving boundary $R(\lambda)$ on which the following conditions are satisfied:

$$W(R, \lambda) = R\,\partial W/\partial R = 0. \tag{4.11}$$

From (4.8) and (4.10) we find that

$$F + dF/d\lambda = \langle\partial W/\partial\lambda\rangle. \tag{4.12}$$

The function $\langle\partial W/\partial\lambda\rangle$ is obtained from (4.10) and (4.11) as follows: integrating (4.10) with respect to ξ in the region $0 \leq \xi \leq R$ we have

$$-\xi\,\partial W/\partial\xi = {}^1/_2\cdot F(R^2 - \xi^2). \tag{4.13}$$

Here, at $\xi = 0$, using (4.5),

$$q(\lambda) = {}^1/_2 FR^2. \quad (4.14)$$

Subsequent integration of (4.13) using (4.14) leads to the result

$$W(\xi, \lambda) = q(\lambda)\left[\ln\frac{R}{\xi} - \frac{1}{2}\left(1 - \frac{\xi^2}{R^2}\right)\right]. \quad (4.15)$$

Differentiating (4.15) with respect to λ and then averaging, we obtain

$$\left\langle\frac{\partial W}{\partial \lambda}\right\rangle = \frac{1}{4} R^{-2}\frac{d}{d\lambda}(qR^2). \quad (4.16)$$

Substituting (4.16) into (4.12) and replacing F and $dF/d\lambda$ with the expressions obtained from (4.14), we find, using (4.6) in the result, the differential equation

$$\frac{d}{d\lambda}(qR^2) = \frac{8(1+e^{-\lambda})}{1+8R^{-2}}. \quad (4.17)$$

Let $\lambda \leq \lambda_1$. On substituting $R(\lambda_1)$ for $R(\lambda)$ in (4.17), in view of the physically obvious inequality $R(\lambda) \leq R(\lambda_1)$, the right-hand side of the equation will be too great. Then, in accordance with S. A. Chaplygin's theorem of differential inequalities for obtaining the upper bound of the function $R(\lambda)$ we have the differential equation

$$\frac{d}{d\lambda}(qR^2) = \frac{8(1+e^{-\lambda})}{1+8R^{-2}(\lambda_1)}.$$

Integrating this equation between the limits $(0, \lambda_1)$, bearing in mind the condition $q(0) = 0$, we find that

$$R^2(\lambda_1) = \frac{8\lambda_1}{1-e^{-\lambda_1}}.$$

Since λ_1 is an arbitrary quantity, the above equation will hold for all λ and gives the upper bound

$$R^2(\lambda) = \frac{8\lambda}{1-e^{-\lambda}}. \quad (4.18)$$

Substituting (4.18) into (4.15) and neglecting terms that are small compared with R, we finally obtain

$$W(\xi_1; \lambda) = \frac{1}{2}(1-e^{-\lambda})\left[\ln\frac{8\lambda}{\xi_1^2(1-e^{-\lambda})} - 1\right], \quad \xi_1^2 = \frac{r_1^2}{\varkappa\tau}.$$

Hence, in accordance with (4.2), we find, converting directly to dimensional variables, that

$$F_\tau(mt) \approx \frac{1}{2}\ln\frac{8t}{1-\exp(-t/\tau)} + $$

$$+ \frac{1}{2}\frac{\tau}{t}\left(1 - \exp\frac{-t}{\tau}\right) - \frac{1}{2}\left(1 + \exp\frac{-t}{\tau}\right).$$

For $mt \geq 100$ and $m\tau \geq 100$, this approximation is very close to the upper bound previously obtained; however, at small values of the indicated parameters it ceases to be satisfactory.

REFERENCES

1. G. I. Barenblatt, Yu. P. Zheltov, and I. N. Kochina, "Basic concepts in the theory of seepage of homogeneous fluids in fractured rocks," PMM, vol. 24, no. 5, 1960.

2. G. I. Barenblatt, "Some boundary value problems for the equation of flow in fractured rocks," PMM, vol. 27, no. 2, 1963.

3. G. Doetsch, Handbuch der Laplace-Transformation [Russian translation], Nauka, 1965.

4. E. B. Chekalyuk, Fundamentals of the Piezometry of Oil and Gas Deposits [in Russian], GITL UkrSSR, 1961.

5. G. Polya and G. Szego, Aufgaben und Lehrsatze aus der Analyse, Vol. 1 [Russian translation], Gostekhizdat, 1965.

6. G. P. Guseinov, Some Problems in the Hydrodynamics of Oil Deposits [in Russian], Azerneshr, 1961.

7. J. E. Warren and P. J. Root, "The behavior of naturally fractured reservoirs," Soc. Petrol. Engnrs. J., vol. 3, no. 3, 1963.

29 February 1967 Baku